Dr.Mahalingam College of Engineering & Technology

Pollachi - 642 003

# **Key for CONTINUOUS ASSESSMENT TEST I**

## Class & Branch: III B.E - CIVIL Max Marks: 50

**Sub.Code & Name: STRUCTURAL ANALYSIS - II Time: 9:30 to 11:00AM**

**Semester: VI Date : 21.01.2011 (FN)**

**Part-A (10×2=20 Marks)**

**Answer all questions:**

1. Define flexibility coefficient

Flexibility co-efficient also known as influence coefficient aij is the generalized deformation (displacement, deflection or rotation) at co-ordinate i due to the generalized unit force (Force or moment) at a co-ordinate j.

1. When stiffness matrix cannot exist?

Stiffness matrix cannot exist for a system coordinates with constrained displacements.

1. Define local and global coordinates.

Structure may consist of many members oriented in different directions. Coordinates oriented with the member axes are called local coordinates. Where as a coordinates used with a common reference axes for all members in the structure is called global coordinates or system system coordinates.

1. What is transformation matrix in the matrix methods of analysis?

In general, transformation matrix relates one co-ordinate system with other co-ordinate system. In the matrix method of analysis, transformation matrix is a matrix that relates the system forces and elemental forces or system displacements and elemental displacements.

1. What does the principle of contragradience suggest in matrix methods of analysis?

The principle of contragradience suggests that the same transformation matrix, which relates system forces and elemental forces, can be used to relate system displacements and elemental displacements and vice versa.

1. What is the degree of static indeterminacies (Total, internal and External indeterminacies) of rigid jointed frame shown in fig. 2.

Internal degree of Indeterminacy I*i* = 3(m-j)+3 = 3(3-4)+3=0

External degree of Indeterminacy I*e* = R-3 = 4-3 =1

Total degree of Indeterminacy I*t* = 3(m-j)+R = 1

1. What is a primary structure?

If the structure is not initially determinate, it has to be made determinate structure by introducing sufficient releases such as hinges and cuts. The structure so reduced to a determinate state is called the primary structure.

1. Define stiffness coefficient

Stiffness co-efficient kij is the force (force or moment) required at co-ordinate i while introducing unit displacement (displacement, deflection or rotation) at j and zero displacement (displacement, deflection or rotation) at all other coordinates.

1. What is kinematic indeterminacy?

Number of independent degrees of freedom required to define the deformed shape of the structure in a unique manner is called kinematic indeterminacy.

1. How sway can be accounted in the matrix stiffness method?

By considering a coordinate for horizontal movement (sway) and including the rotation in transformation matrix due to that sway, the effect of sway can be added to the stiffness matrix. Then, the matrix stiffness method shall be followed.

### Part-B (3\*10=30 Marks)

**Answer any Three**

* 1. Explain the steps involved in of flexibility matrix method and stiffness matrix method side by side in the form of tabular column. (10)

|  |  |  |
| --- | --- | --- |
| STEPS | FLEXIBILITY MATRIX METHOD | STIFFNESS MATRIX METHOD |
|  | Static indeterminacy is to be findout | Kinematic indeterminacy is to be findout. |
|  | Primary structure is to be made by releasing redundants member with hinges and cut and with unknown forces {F0} |  |
|  | Elemental (internal) coordinates ( for {P}, {δ} ) and system coordinates ( for {F}, {*u*} ) are to be determined. | Elemental (internal) coordinates ( for {P}, {δ} ) and system coordinates (for {F}, {*u*} ) for each kinematic degree of freedom are to determined |
|  | Find Fixed end moments {FEM} due to off joint loads and equivalent joint forces {Pe} for off joint loads which is {Pe} = - {FEM} | Fixed end moments {FEM} due to off joint loads is to be found Equivalent elemental forces at joints {Pe} for off joint loads is {Pe} = - {FEM} |
|  | A matrix [bs] relating elemental forces to the known system forces {Fs} is to formed | Transformation matrix [β] is to be found out that relates elemental coordinates and system coordinates (i.e){P} = [β] {F} |
|  | A matrix [bo] relating elemental forces to the unknown system forces {Fo} is to formed. | {Fs} system load due to the off joint loads is to be found using {Fs}= [β]T {Pe} |
|  | Then transformation matrix [b] = [ [bs] [bo]].  | Find {Fj}, if any, which are external moments or forces acting exclusively at jointsTotal system force {F} = {Fs}+{Fj} |
|  | Flexibility matrices [*α1*], [*α2*], [*α3*] etc for elements based on their types and assembled flexibility matrix [*α*] for the structure is to be formed. | Stiffness matrices [*k1*], [*k2*], [*k3*] etc for elements based on their types and assembled stiffness matrix [*k*] for the structure is to be formed. |
|  | [a00] = [bo]T [*α*] [bo] and its inverse [a00]-1 are to be obtained | System stiffness matrix [K] is to be formed using [K] = [β]T [*k*] [β]  |
|  | [a0S] = [bo]T [*α*] [bS] is to be obtained | $\left[K\right]^{-1}$ is to be found |
|  | Now, Unknown forces can be found using {F0} = -[a00]-1 [a0S] {Fs} | System displacement {*u*} can be found using {*u*} = $\left[K\right]^{-1} ${F} |
|  | All the system forces now are known and they are $\left\{F\right\}=\left\{\begin{matrix}\left\{F\_{s}\right\}\\\left\{F\_{o}\right\}\end{matrix}\right\}$ | Elemental displacements can be found using{δ}= [β] {*u*} |
|  | After finding system forces, elemental forces can be found for the external forces {F} as{P'} = [b] {F} .  | After finding elemental displacements, elemental forces can be found from{P'} = [k] {δ} |
|  | {P'} is the elemental forces (mostly fixed end moments) due to force {F} and hence adding {P'} and {FEM} gives the final fixed end moments. {P*f*} = {P'} + {FEM} or {P*f*} = {P'} – {Pe}From these values BMD can be drawn and SFD also can be drawn after finding reactions. | {P'} is the elemental forces (mostly fixed end moments) due to force {F} and hence adding {P'} and {FEM} gives the final fixed end moments. {P*f*} = {P'} + {FEM} or {P*f*} = {P'} – {Pe}From these values BMD can be drawn and SFD also can be drawn after finding reactions |

* 1. Generate flexibility matrix [a] of the structure with three coordinates indicated in the Fig. 1 by the direct method. (10)

0.8L

L

EI=constant

A

B

C

D

0.7L

E

3

1

**Fig. 1**

2

Direct Method.

Flexibility coefficients for system co-ordiate 1.

Apply unit force at co-ordinate 1 and measure

Displacements corresponding to 1, 2 and 3.

Rotation at 1 (a11) = rotation between AC+

 rotation between CE

Rotation at 2 (a21) = rotation at C

Displacement 3 (a31) = rotation at B\*L

$$a\_{11}=\frac{1}{EI}\left(2.5L\right)$$

$a\_{21}=\frac{1}{EI}\left(1.5L\right)$

A

B

C

D

E

1

$a\_{31}=\frac{1}{EI}\left(0.7L\right)$\*L

Flexibility coefficients for system co-ordiate 2.

Apply unit force at co-ordinate 2 and measure

Displacements corresponding to 1, 2 and 3.

Rotation at 1 (a12) = rotation between AC

A

B

C

D

E

3

A

B

C

D

E

2

Rotation at 2 (a22) = rotation at C

Displacement 3 (a32) = rotation at B\*L

$$a\_{12}=\frac{1}{EI}\left(1.5L\right)$$

$$a\_{22}=\frac{1}{EI}\left(1.5L\right)$$

$a\_{32}=\frac{1}{EI}\left(0.7L\right)$\*L

Flexibility coefficients for system co-ordiate 3.

Apply unit force at co-ordinate 3 and measure

Displacements corresponding to 1, 2 and 3.

Rotation at 1 (a13) = rotation at B

Rotation at 2 (a23) = rotation at AB

Displacement 3 (a33) = rotation at B\*L+

 displacement between BD

$a\_{13}=\frac{1}{EI}\*L\*\left(0.7L\right)$

% matlab code for indirect method for this problem

b=[-1 1 -1 1 -1 1 0 0;-1 1 -1 1 0 0 0 0;

 -L L 0 0 0 0 -L 0]'

EI=sym('EI','real');

L=sym('L','real');

L1=0.7\*L;

L2=0.8\*L;

L3=L;

L4=L;

al1=L1/(6\*EI)\*[2 -1;-1 2]

al2=L2/(6\*EI)\*[2 -1;-1 2]

al3=L3/(6\*EI)\*[2 -1;-1 2]

al4=L4/(6\*EI)\*[2 -1;-1 2]

alpha=[al1 zeros(2,6);zeros(2,2) al2 zeros(2,4);

 zeros(2,4) al3 zeros(2,2);zeros(2,6) al4]

a=b' \* alpha \* b

$$a\_{23}=a\_{13}$$

$a\_{33}=\frac{1}{EI}L\*\left(0.7L\right)$\*L+$\frac{L^{3}}{3EI}$

Thus, Flexibility matrix [a] is

$$[a]=\frac{L}{EI}\left[\begin{matrix}2.5&1.5&0.7L\\1.5&1.5&0.7L\\0.7L&0.7L&1.033L^{2}\end{matrix}\right]$$

* 1. Analysis the portal frame shown in Fig. 2 using the flexibility matrix method if IAB=2Io; IBC = Io ; ICD=Io and E is constant. (10)

**Fig. 2**

4 m

6m

A

D

C

B

100 kN

**Step 1 – Identifying Degree of static indeterminacy**

**and making primary structure**

Degree of Static determinacy is 1 (External).

Considering vertical reaction at D as redundanct and

replacing it by unknown force Fo1 the Primary structure is

obtained as follows:

A

D

C

B

Fs1=100 kN

Fo1

Now there are two forces Fs1  and Fo1 acting in the primary structure which is equivalent of the given structure and system of forces. FEM={0}.

 **Step 2 – Formation of transformation matrix [bs], [bo] and [b]**

A

D

C

B

1

2

4

3

5

6

Elemental forces

[bs] = $\left[\begin{matrix}\begin{matrix}-4\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right] $ ; [b0]= $\left[\begin{matrix}\begin{matrix}6\\-6\end{matrix}\\\begin{matrix}6\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right] $ and [b] = $\left[\begin{matrix}\begin{matrix}-4\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\begin{matrix}\begin{matrix}6\\-6\end{matrix}\\\begin{matrix}6\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right]$

**Step 3 – Element flexibility matrices and assembled flexibility matrix [α]**

$Flexibility matrix for beam element is \frac{L}{6EI}\left[\begin{matrix}2&-1\\-1&2\end{matrix}\right]$ ;

 $α\_{1}=\frac{1}{EI}\left[\begin{matrix}0.667&-0.333\\-0.333&0.667\end{matrix}\right]; α\_{2}=\frac{1}{EI}\left[\begin{matrix}2&-1\\-1&2\end{matrix}\right]; α\_{3}=\frac{1}{EI}\left[\begin{matrix}1.333&-0.667\\-0.667&1.333\end{matrix}\right]$

$$\left[α\right]=\frac{1}{EI}\left[\begin{matrix}\begin{matrix}0.667&-0.333\\-0.333&0.667\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix} 2&-1\\-1&2\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}1.333&-0.667\\-0.667&1.333\end{matrix}\end{matrix}\right]$$

**Step 4 – Find [a00] and [a0s]**

$$a\_{00}=\left[b\_{0}\right]^{T} [α]\left[b\_{0}\right]=\left[\begin{matrix}\begin{matrix}6&-6&6\end{matrix}&\begin{matrix}0&0&0\end{matrix}\end{matrix}\right]\frac{1}{EI}\left[\begin{matrix}\begin{matrix}0.667&-0.333\\-0.333&0.667\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix} 2&-1\\-1&2\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}1.333&-0.667\\-0.667&1.333\end{matrix}\end{matrix}\right]\left[\begin{matrix}\begin{matrix}6\\-6\end{matrix}\\\begin{matrix}6\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right]$$

$$a\_{00}=\frac{144}{EI}$$

$$a\_{0s}=\left[b\_{0}\right]^{T} [α]\left[b\_{s}\right]=\left[\begin{matrix}\begin{matrix}6&-6&6\end{matrix}&\begin{matrix}0&0&0\end{matrix}\end{matrix}\right]\frac{1}{EI}\left[\begin{matrix}\begin{matrix}0.667&-0.333\\-0.333&0.667\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix} 2&-1\\-1&2\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}1.333&-0.667\\-0.667&1.333\end{matrix}\end{matrix}\right]\left[\begin{matrix}\begin{matrix}-4\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right]$$

$$a\_{0s}=\frac{-24}{EI}$$

**Step 5 – Find** $[a\_{00}]^{-1}$

$$[a\_{00}]^{-1}=\frac{EI}{144}$$

**Step 6 – Finding unknown forces and system forces.**

Unknown forces $\left\{F\_{0}\right\}=$ $[a\_{00}]^{-1 }\left[a\_{0s}\right]\left\{F\_{s}\right\}=-\frac{EI}{144}×\frac{-24}{EI}×100=\left\{16.67\right\}$

Hence system forces $\left\{F\right\}= \left\{\begin{matrix}100\\16.67\end{matrix}\right\}$

**Step 7 – Elemental forces due to {F}**

{P’}=[b]{F} = $\left[\begin{matrix}\begin{matrix}-4\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\begin{matrix}\begin{matrix}6\\-6\end{matrix}\\\begin{matrix}6\\0\end{matrix}\\\begin{matrix}0\\0\end{matrix}\end{matrix}\right]\left\{\begin{matrix}100\\16.67\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}-300\\-100\\100\end{matrix}\\\begin{matrix}0\\0\\0\end{matrix}\end{matrix}\right\}$

**Step 8 – Final elemental forces {Pf}**

{Pf}={P’}+{FEM} = $\left\{\begin{matrix}\begin{matrix}-266.67\\-133.33\\133.33\end{matrix}\\\begin{matrix}0\\0\\0\end{matrix}\end{matrix}\right\}$

* 1. Using the matrix flexibility method, find the member forces in the pin-jointed truss shown in Fig. 3. Cross sectional areas of members are : 500mm2 and E=200kN/mm2. (10)

3 m

A

B

C

D

Fs1=100 kN

 4m

45º

F01

F01

36.87º

**Fig. 3**

3 m

A

B

C

D

100 kN

 4m

45º

5 m

**Step 1 – Identifying Degree of**

**static indeterminacy and**

**making primary structure**

Degree of Static

 determinacy is 1 (Internal).

Considering the member BD

as redundanct and replacing it

by unknown force Fo1 the Primary

structure is obtained as follows:

Now there are two forces Fs1  and Fo1 acting in the primary structure which is equivalent of the given structure and system of forces.

 **Step 2 – Formation of transformation matrix [bs], [bo] and [b]**

Apply unit force at only Fs1 coordinate and find the member forces

Reactions VB = -0.707\*3/4= -0.53kN ; VA = 1.237kN and HA = 0.707 kN

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Joint | Member | Force Diagram | $$\sum\_{}^{}F\_{x}=0$$ | $$\sum\_{}^{}F\_{y}=0$$ | $$F\_{i}$$ |
| D | 1 | 45º1kN | F2 –cos(45) =0 | F1+sin(45) =0 | -0.707 |
| 2 | 0.707 |
| C | 2(known) | 0.70736.87º | -0.707-F5 Cos(36.87)=0 | +F3+ F5 Sin(36.87)=0 |  |
| 3 | 0.53 |
| 5 | -0.884 |
| B | VB(known) | 0.530.53 | -F4=0 | -0.53+0.53=0 |  |
| 3(known) |  |
| 4 | 0 |

A

B

C

D

1kN

1kN

36.87º

[bs] = $\left[\begin{matrix}\begin{matrix}-0.707\\0.707\end{matrix}\\\begin{matrix}0.53\\0\end{matrix}\\\begin{matrix}-0.884\\0\end{matrix}\end{matrix}\right] $

Apply unit force at only F01 coordinate and find the member forces

Reactions VB = 0 ; VA = 0kN and HA = 0 kN

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Joint | Member | Force Diagram | $$\sum\_{}^{}F\_{x}=0$$ | $$\sum\_{}^{}F\_{y}=0$$ | $$F\_{i}$$ |
| D | 1 | 1kN36.87º | F2 +cos(36.87) =0 | F1+sin(36.87) =0 | -0.6 |
| 2 | -0.8 |
| C | 2(known) | 0.836.87º | 0.8-F5 Cos(36.87)=0 | +F3+ F5 Sin(36.87)=0 |  |
| 3 | -0.6 |
| 5 | 1.0 |
| B | VB(known) | 1kN36.87º0.6 | -F4- cos (36.87)=0 |  |  |
| 3(known) |  |
| 4 | -0.8 |

[b0] = $\left[\begin{matrix}\begin{matrix}-0.6\\-0.8\end{matrix}\\\begin{matrix}0.60\\-0.80\end{matrix}\\\begin{matrix}1\\1\end{matrix}\end{matrix}\right] $ and [b] = $\left[\begin{matrix}\begin{matrix}-0.707\\0.707\end{matrix}\\\begin{matrix}0.53\\0\end{matrix}\\\begin{matrix}-0.884\\0\end{matrix}\end{matrix} \begin{matrix}\begin{matrix}-0.6\\-0.8\end{matrix}\\\begin{matrix}0.60\\-0.80\end{matrix}\\\begin{matrix}1\\1\end{matrix}\end{matrix}\right]$

**Step 3 – Element flexibility matrices and assembled flexibility matrix [α]**

AE=500x200=1x105 for all members. Element flexibility matrix is [L/AE]

Hence [α]=$ \frac{1}{10^{5}} \left[\begin{matrix}\begin{matrix}3&0&0\\0&4&0\\0&0&3\end{matrix}&\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}\\\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}&\begin{matrix}4&0&0\\0&5&0\\0&0&5\end{matrix}\end{matrix}\right]$

**Step 4 – Find [a00] and [a0s]**

$$a\_{00}=\left[b\_{0}\right]^{T} [α]\left[b\_{0}\right]=\left[-0.\begin{matrix}\begin{matrix}6&-0.8&0.6\end{matrix}&\begin{matrix}-0.8&1&1\end{matrix}\end{matrix}\right]\frac{1}{10^{5}} \left[\begin{matrix}\begin{matrix}3&0&0\\0&4&0\\0&0&3\end{matrix}&\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}\\\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}&\begin{matrix}4&0&0\\0&5&0\\0&0&5\end{matrix}\end{matrix}\right]\left[\begin{matrix}\begin{matrix}-0.6\\-0.8\end{matrix}\\\begin{matrix}0.6\\-0.8\end{matrix}\\\begin{matrix}1\\1\end{matrix}\end{matrix}\right]$$

$$a\_{00}=1.7280×10^{-4}$$

$$a\_{0s}=\left[b\_{0}\right]^{T} [α]\left[b\_{s}\right]=\left[-0.\begin{matrix}\begin{matrix}6&-0.8&0.6\end{matrix}&\begin{matrix}-0.8&1&1\end{matrix}\end{matrix}\right]\frac{1}{10^{5}} \left[\begin{matrix}\begin{matrix}3&0&0\\0&4&0\\0&0&3\end{matrix}&\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}\\\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}&\begin{matrix}4&0&0\\0&5&0\\0&0&5\end{matrix}\end{matrix}\right]\left[\begin{matrix}\begin{matrix}-0.707\\0.707\end{matrix}\\\begin{matrix}0.53\\0\end{matrix}\\\begin{matrix}-0.884\\0\end{matrix}\end{matrix}\right] $$

$$a\_{0s}= -4.4558×10^{-5}$$

**Step 5 – Find** $[a\_{00}]^{-1}$

$$[a\_{00}]^{-1}= 5.7870×10^{3}$$

**Step 6 – Finding unknown forces and system forces.**

Unknown forces $\left\{F\_{0}\right\}=$ $[a\_{00}]^{-1 }\left[a\_{0s}\right]\left\{F\_{s}\right\}=-\left(5.7870×10^{3}\right)×(-4.4558×10^{-5})×100=\left\{25.7859\right\}$

Hence system forces $\left\{F\right\}= \left\{\begin{matrix}100\\25.7859\end{matrix}\right\}$

**Step 7 – Final Elemental forces**

{P}=[b]{F} = $\left[\begin{matrix}\begin{matrix}-0.707\\0.707\end{matrix}\\\begin{matrix}0.53\\0\end{matrix}\\\begin{matrix}-0.884\\0\end{matrix}\end{matrix} \begin{matrix}\begin{matrix}-0.6\\-0.8\end{matrix}\\\begin{matrix}0.60\\-0.80\end{matrix}\\\begin{matrix}1\\1\end{matrix}\end{matrix}\right]\left\{\begin{matrix}100\\25.7859\end{matrix}\right\}=\left\{\begin{matrix}\begin{matrix}-86.17\\50\\68.47\end{matrix}\\\begin{matrix}-20.63\\-62.61\\25.79\end{matrix}\end{matrix}\right\}$

* 1. Analyse the frame shown in fig. 4 by matrix stiffness method. (10)

**Fig. 4**

4 m

40kN/m

10m

 8m

A

D

C

B

8 m

I

I

2I

E = constant

**Step 1 – Identification of system coordinates**

In Joint B and Joint C will have a rotation. Since the given

frame is an unsymmetric, there will be sway. Hence

additionally a lateral movement will be introduced at B. Thus

system co-ordinates are as follows:

A

D

C

B

1

2

3

D

A

C

B

1

2

3

4

6

5

Elemental Co-ordinates

**Step 2 – Fixed End moments and element forces at joints.**

FEM={0 0 -213.33 213.33 0 0}’

Pe={0 0 213.33 -213.33 0 0}’

**Step 3 – Formation of transformation matrix [β]**

[β] = $\left[\begin{matrix}\begin{matrix}-0.25\\-0.25\end{matrix}\\\begin{matrix}0\\0\end{matrix}\\\begin{matrix}-0.125\\-0.125\end{matrix}\end{matrix} \begin{matrix}\begin{matrix}0&0\\1&0\end{matrix}\\\begin{matrix}1&0\\0&1\end{matrix}\\\begin{matrix}0&1\\0&0\end{matrix}\end{matrix}\right]$

**Step 4 – System forces {Fs} equivalent to the elemental forces**

{Fs} = [β]T {Pe}== $\left[\begin{matrix}\begin{matrix}-0.25&-0.25&0\\0&1&1\\0&0&0\end{matrix}&\begin{matrix}0&-0.125&-0.125\\0&0&0\\1&1&0\end{matrix}\end{matrix}\right]\left\{\begin{matrix}\begin{matrix}0\\0\\213.33\end{matrix}\\\begin{matrix}-213.33\\0\\0\end{matrix}\end{matrix}\right\}= \left\{\begin{matrix}0\\213.33\\-213.33\end{matrix}\right\}$

Externally applied system forces at joints are zero since there is no external force is found in the problem {Fj}={0}

{F}={Fs}+{Fj} = {Fs}=$\left\{\begin{matrix}0\\213.33\\-213.33\end{matrix}\right\}$

**Step 5 – Element Stiffness matrices and Assembled stiffness matrix**

Element stiffness matrix for beam element is $\frac{4EI}{L}\*\left[\begin{matrix}1&0.5\\0.5&1\end{matrix}\right]$

$k\_{1}=\frac{4EI}{4}\*\left[\begin{matrix}1&0.5\\0.5&1\end{matrix}\right]$ ; $k\_{2}=\frac{4EI}{8}\*\left[\begin{matrix}1&0.5\\0.5&1\end{matrix}\right]$; $k\_{3}=\frac{4E(2I)}{8}\*\left[\begin{matrix}1&0.5\\0.5&1\end{matrix}\right]$

Hence assembled stiffness matrix [k] is

$$[k]=EI\left[\begin{matrix}\begin{matrix}1&0.5\\0.5&1\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0.5&0.25\\0.25&0.5\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}1&0.5\\0.5&1\end{matrix}\end{matrix}\right]$$

**Step 6 – System Stiffness Matrix [K] and its inverse**

[K] = [β]T [k] [β]= $\left[\begin{matrix}\begin{matrix}-0.25&-0.25&0\\0&1&1\\0&0&0\end{matrix}&\begin{matrix}0&-0.125&-0.125\\0&0&0\\1&1&0\end{matrix}\end{matrix}\right] EI\left[\begin{matrix}\begin{matrix}1&0.5\\0.5&1\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0.5&0.25\\0.25&0.5\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}1&0.5\\0.5&1\end{matrix}\end{matrix}\right]$ $\left[\begin{matrix}\begin{matrix}-0.25\\-0.25\end{matrix}\\\begin{matrix}0\\0\end{matrix}\\\begin{matrix}-0.125\\-0.125\end{matrix}\end{matrix} \begin{matrix}\begin{matrix}0&0\\1&0\end{matrix}\\\begin{matrix}1&0\\0&1\end{matrix}\\\begin{matrix}0&1\\0&0\end{matrix}\end{matrix}\right]$

[K]=EI $\left[\begin{matrix}0.2344&-0.375&-0.1875\\-0.375&1.5&0.25\\-0.1875&0.25&1.5\end{matrix}\right]$

$$\left[K\right]^{-1}=\frac{1}{EI}\left[\begin{matrix}7.698&-1.814&0.660\\1.814&1.113&0.041\\0.660&0.041&0.742\end{matrix}\right]$$

**Step 7 – System displacements {u} and elemental displacements {δ}**

$\left\{u\right\}= \left[K\right]^{-1 }\left\{F\right\}$= $\frac{1}{EI}\left[\begin{matrix}7.698&-1.814&0.660\\1.814&1.113&0.041\\0.660&0.041&0.742\end{matrix}\right]\left\{\begin{matrix}0\\213.33\\-213.33\end{matrix}\right\}$

$$\left\{u\right\}=\frac{1}{EI}\left\{\begin{matrix}246.3192\\ 228.7249\\-149.5509\end{matrix}\right\}$$

$\left\{δ\right\}= \left[β\right]\left\{u\right\}$=$\left[\begin{matrix}\begin{matrix}-0.25\\-0.25\end{matrix}\\\begin{matrix}0\\0\end{matrix}\\\begin{matrix}-0.125\\-0.125\end{matrix}\end{matrix} \begin{matrix}\begin{matrix}0&0\\1&0\end{matrix}\\\begin{matrix}1&0\\0&1\end{matrix}\\\begin{matrix}0&1\\0&0\end{matrix}\end{matrix}\right]\frac{1}{EI}\left\{\begin{matrix}246.3192\\ 228.7249\\-149.5509\end{matrix}\right\}=\frac{1}{EI}\left\{\begin{matrix}\begin{matrix}-61.58\\167.15\\228.72\end{matrix}\\\begin{matrix}-149.55\\-180.34\\-30.79\end{matrix}\end{matrix}\right\}$

**Step 8 – Elemental forces {P’}**

$$\left\{P^{'}\right\}= \left[k\right]\left\{δ\right\}= EI\left[\begin{matrix}\begin{matrix}1&0.5\\0.5&1\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0.5&0.25\\0.25&0.5\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}\\\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}0&0\\0&0\end{matrix}&\begin{matrix}1&0.5\\0.5&1\end{matrix}\end{matrix}\right]\frac{1}{EI}\left\{\begin{matrix}\begin{matrix}-61.58\\167.15\\228.72\end{matrix}\\\begin{matrix}-149.55\\-180.34\\-30.79\end{matrix}\end{matrix}\right\}$$

$=\left\{\begin{matrix}\begin{matrix}22.0\\136.36\\76.97\end{matrix}\\\begin{matrix}-17.59\\-195.74\\-120.96\end{matrix}\end{matrix}\right\} $

**Step 9 – Final Fixed End moments {Pf}**

$$\left\{P^{f}\right\}=\left\{P^{'}\right\}+\left\{FEM\right\}$$

$=\left\{\begin{matrix}\begin{matrix}22.0\\136.36\\76.97\end{matrix}\\\begin{matrix}-17.59\\-195.74\\-120.96\end{matrix}\end{matrix}\right\}+\left\{\begin{matrix}\begin{matrix}0\\0\\-213.33\end{matrix}\\\begin{matrix}213.33\\0\\0\end{matrix}\end{matrix}\right\}$= $\left\{\begin{matrix}\begin{matrix}22\\136.36\\-136.36\end{matrix}\\\begin{matrix}195.74\\-195.74\\-120.96\end{matrix}\end{matrix}\right\}$

**Bending moment diagram**

* 1. A pin-jointed frame consists of three members connected as shown in fig. 5. Compute the internal forces in members using matrix displacement method. Take E=2x105 N/mm2 for all members. Area of all members are equal, A=500mm2. (10)

**Step 1 – Identification of system coordinates**

A

B

**O**

50kN

3m

4m

C

5m

Degrees of freedom is 2 at joint O. One in vertical and

the other in horizontal directions.

5m

**Step 2 –Elemental coordinates.**

6.403m

One coordinate for each member in the axial direction.

**Step 3 – Formation of transformation matrix [β]**

[β] = $\left[\begin{matrix}0.8&0.6\\1&0\\0.625&-0.781\end{matrix}\right]$

A

B

**O**

50kN

3m

4m

C

5m

**Step 4 – System forces {Fs} equivalent to the elemental forces**

{F}= $\left\{\begin{matrix}50\\0\end{matrix}\right\}$

A

B

**O**

50kN

3m

4m

C

5m

**Step 5 – Element Stiffness matrices and Assembled stiffness matrix**

Element stiffness matrix for bar element is $\frac{AE}{L}$

AE is same foe all members = 2x105 x 500 = 108 N = 105 kN

$k\_{1}=\frac{10^{5}}{5}$; $k\_{2}=\frac{10^{5}}{4}$; $k\_{1}=\frac{10^{5}}{6.403}$;

Hence assembled stiffness matrix [k] is

$$[k]=10^{5}\left[\begin{matrix}0.2&0&0\\0&0.25&0\\0&0&0.156\end{matrix}\right]$$

**Step 6 – System Stiffness Matrix [K] and its inverse**

[K] = [β]T [k] [β]= $\left[\begin{matrix}0.8&1&0.625\\0.6&0&-0.781\end{matrix}\right]10^{5}\left[\begin{matrix}0.2&0&0\\0&0.25&0\\0&0&0.156\end{matrix}\right] \left[\begin{matrix}0.8&0.6\\1&0\\0.625&-0.781\end{matrix}\right]$

[K] = $10^{4}\left[\begin{matrix}4.3894&0.1985\\0.1985&1.6715\end{matrix}\right]$

$$\left[K\right]^{-1}=10^{-4}\left[\begin{matrix}0.2291&-0.0272\\-0.0272&0.6015\end{matrix}\right]$$

**Step 7 – System displacements {u} and elemental displacements {δ}**

$\left\{u\right\}= \left[K\right]^{-1 }\left\{F\right\}$= $10^{-4}\left[\begin{matrix}0.2291&-0.0272\\-0.0272&0.6015\end{matrix}\right]\left\{\begin{matrix}50\\0\end{matrix}\right\}$

$$\left\{u\right\}=10^{-4}\left\{\begin{matrix}1.145\\-0.136\end{matrix}\right\}$$

$\left\{δ\right\}= \left[β\right]\left\{u\right\}$=$\left[\begin{matrix}0.8&0.6\\1&0\\0.625&-0.781\end{matrix}\right]10^{-4}\left\{\begin{matrix}1.145\\-0.136\end{matrix}\right\}=10^{-4}\left\{\begin{matrix}0.835\\1.145\\0.822\end{matrix}\right\}$

**Step 8 – Final Elemental forces {Pf}**

 $\left\{P^{f}\right\}= \left[k\right]\left\{δ\right\}= 10^{4}\left[\begin{matrix}0.2&0&0\\0&0.25&0\\0&0&0.156\end{matrix}\right]10^{-4}\left\{\begin{matrix}0.835\\1.145\\0.822\end{matrix}\right\}=\left\{\begin{matrix}16.692\\28.632\\12.824\end{matrix}\right\}$

5m

C

3m

B

A

28.632kN

12.824kN

16.692kN

4m

**O**

50kN

--------------------------------- End ---------------------------------